

Mathematical Modeling within a Technology Based Learning Environment: Some Principles for Adaptive Instruction

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Abstract: This paper describes a set of research based instructional plans being developed by a multidisciplinary team of researchers at the University of Nebraska within a National Science Foundation Proof-of-Concept Grant. The project is seeking to establish a prototype effort for teaching mathematical modeling within a technology based learning environment, which includes components of adaptive instruction for the student. Seven curriculum based design principles for the adaptive instruction that are being followed in the development of the project are described in the paper as they relate to research on effective mathematics instruction, and in particular the enhancement of mathematical modeling activities.

Why Mathematical Modeling?

This paper describes seven design principles, and related development plans, being integrated into a curriculum design project within the context of a NSF Proof-of-Concept grant. The project is targeting mathematical modeling as a content area within the project, because mathematical modeling is both an important topic in today's mathematics classroom, and an unusually difficult process to teach in the traditional classroom. Mathematical modeling can be defined as a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyses (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model (Swetz & Hartzler, 1991).. It is essentially a systematic generalization process, where the mathematical model (such as a mathematical expression or algebraic formula) attempts to describe the mathematical relationships for a group of problems or situations, and is refined over a period of time with additional testing or use of the model.

Mathematical modeling can be difficult to teach in traditional formats, such as lecture, and often requires considerable student involvement. Part of the difficulty in the instruction of mathematical modeling, is that considerable flexibility and feedback is often needed to work with the student (Smith, 1997). As a student's understanding evolves, their conceptual model may go through many different evolutions, hopefully becoming more refined over a period of time, and with more instruction and feedback. Often, if a formula can be used to represent the model, the formula evolution itself may somewhat represent the evolution in the modeling process. Mathematical modeling is in essence a "scientific inquiry" process for mathematics, and can be thought of as being undertaken in a series of four stages, which become cyclical as the model refines. Four stages can be considered within the mathematical modeling process typically undertaken (Swetz & Hartzler, 1991). These stages include: Stage 1 - Observing and Discerning (observe the phenomenon or problem); Stage 2 - Conjecturing (proposing a mathematical or symbolic representation of the problem); Stage 3 - Applying Mathematical Analysis (converting relationships within the data based model to mathematical equations or expressions); and Stage 4 - Interpreting Results (test the model, and obtain results and interpret them in the context of the original problem).

The Teaching of Mathematical Modeling

Mathematical modeling is a key process for the complex problem solving that takes place in businesses and industry, and applied mathematics in engineering, as well as other fields. Due to the applied focus of mathematical modeling, there are lots of real life problems lending themselves to mathematical modeling, such as predicting wildlife populations, costs of long distance phone calls, irrigation flow rates, and even the fastest line to enter in a check out stand at a grocery. From a classroom perspective, these problems often lend themselves well to interactive multimedia and technology based instruction, where a simulation might be used as part of the instruction, as well as a systematic questioning process involving student dialogue or discussion. The use of such interactive activity within an electronic course format can be a powerful mechanism for building the knowledge base of students, as well as assessing the individual skills of students (Richards, Barker, Meng Tan, Hudson, and Beachman, 1997); and such work has already been successfully integrated into limited knowledge transfer systems.

The effective instruction of mathematical modeling within a classroom context or related course format is often built upon several important assumptions or considerations (Ostler & Grandgenett, 1999). These include the following: 1) Students have some control over how they approach the problem, 2) Good modeling activities are adaptable to many different ability levels, 3) Good modeling activities are scalable to different grade levels, 4) Problem solving and mathematical modeling are retained as different but related processes, 5) Mathematical modeling is used to focus primarily on the general case, and 6) The mathematical modeling is assessed carefully within the learning process, since even a poor model may build student understanding as it is tested, and then discarded by the student.

Adapting Instruction for Effective Mathematical Modeling

The designed project undertaken within this NSF Proof-of-Concept project is initially targeting the instructional topic of acceleration, which is a common topic covered in a variety of developmental mathematics and science courses. Especially relevant to the choice of developmental math for the prototype design effort is the fact that developmental math courses are taught in 72 percent of four-year institutions of higher education in the U.S., and nearly every (99 percent) two-year colleges. Students in these remedial classes are often nearly on their own, left to work their way through a textbook with only a graduate student instructor available to answer questions and offer assistance. Many universities, including the University of Nebraska system, are not able or willing to use scarce and expensive instructors in what are essentially seen as remedial courses. Using the instructional approach being planned within the design process, if expanded, would provide the advantage of more interactive and personalized instruction than what is usually now available in such developmental math courses.

The overall vision for the technology based learning environment that is being designed in this NSF Proof-of-Concept endeavor is one which is consistent with the vision of new technology based resources as recommended by documents such as the 1996 NSF document "Shaping the Future: New Expectations for Undergraduate Education in Science, Mathematics, Engineering and Technology." In addition, the use of a technologies that assist individual learners with different instructional styles and educational opportunities is a key design consideration. Within this context, the project is also striving to develop technology-based applications which can help illustrate the utility of "learning communities" within such instruction, that can provide a flexible and extended learning process which might take place both inside and outside of the traditional classroom walls. The possible contribution to a student's individual learning process, as represented by such learning communities is just now being fully recognized. As described by the SRI International Center for Technology in Learning, in the report Future Visions by the United States Congress Office of Technology Assessment, the educational potential and utility is considerable: "the effective use of this technology could alter the relationships between homes, schools, and workplaces and in so doing assist the creation of new kinds of communities --- communities that have learning and teaching at their core and use digital technologies to foster higher levels of community participation, enable deeper levels of cognitive and social engagement, and structure new kinds of relationships that support education." (From Future Visions, Kozman and Grant, 1995, pg. 121)

In order to ensure that the project is designed with the individual student learning process as the priority consideration, research-based educational design principles are being carefully integrated into all aspects of the

project development. In essence, the educational vision for the adaptive instruction to be used within the project is such that the instructional process will embody the following seven design principles.

Design Principle 1) The adaptive instruction will seek to be a use of technology that helps students learn through involvement with real life problems, real life data, and true examples of mathematical modeling as they apply to today's world.

The use of real life problems, data, and tools within the context of technology based mathematics instruction has long been recognized as a beneficial contribution to student learning (Corbat, 1985). The availability of the Internet has expanded the teacher's selection of such modeling resources and software, which are now available to a much greater extent than they were even five years ago (Harvey and Charnitski, 1998). It is this new networking capability that promises to provide teachers everywhere with an enhanced opportunity to incorporate mathematical modeling activities into existing curriculum and to give their students a chance to undertake modeling activities that are fairly realistic. For example, students might examine actual census data using new statistical tools to create their own predictive model of a societal trend. In addition, such new resources and tools also suggest the potential for a wider use of mathematical examples across grade levels, with greater flexibility in how a student might approach a mathematical modeling activity. For instance, real life examples related to optimization were normally not taught until Calculus, but with the appropriate graphing technologies, students at lower grade levels can learn to interpret and build mathematical generalizations based upon graphical information as well as the traditional calculus approach. This exposure in the lower grades (i.e. algebra or geometry) would set the stage for much more meaningful problem solving and mathematical modeling when the same students reach Calculus and study optimization as a formal topic.

Design Principle 2) The adaptive instruction will seek to actively rather than passively involve students, in deep conceptual questions and encourage them to be both dynamic and flexible in their thinking and problem solving.

A fundamental instructional idea behind mathematical modeling is that students, through modeling activities, discover patterns and consistencies in data that will allow them to test, refine, and build generalizations by creating a "mathematical machine" which represents a particular situation (Smith, 1997). This "machine" would provide them with a means for conjectures and predictions that might be tested using data sets, or systematic trials. Thus, the mathematical modeling process by a student typically goes through several modifications or refinements in order to produce a model which is more accurate, faster, or efficient. The creation of such a mathematical machine by a student, and its testing and refinement, is typically a very interactive process. Such systematic thinking within the mathematics field by a student is similar to they might undertake using the scientific method within a science class, and parallels that process closely. It also helps students understand that true mathematical application is much more than the mere routine application of formulas and strategies that they may have experienced in some mathematics instruction.

Design Principle 3) The adaptive instruction system will seek to be an additional resource to teachers and classrooms, rather than a replacement for these valuable assets to student understanding.

The design project being undertaken in this effort is striving to enhance rather than replace the important synergy that often happens between a teacher and student in the learning process. Within this context, the technology being designed seeks to facilitate the shared thinking between a student and their teacher, or a student and other students. The technology should also help organize and coordinate the technology based resources available to a student within the classroom environment, in addition to the teacher. For example, students often don't know what technology based tools might be useful for helping with a particular mathematical modeling task. How might they use a spreadsheet to help examine their data? How might they test some of their evolving ideas within a mathematical simulation? Where might they go on the web for additional information on a modeling topic? This third design principle then seeks to permit the student as much control as possible within the learning process, and still help the student structure their thinking, and coordinate the access of their classroom resources, so that they can work more efficiently and use their resources more effectively.

Design Principle 4) The adaptive instruction will seek to enhance human interaction, by connecting students more effectively with the teacher, their peers (other students), and appropriate mentors (professionals) during the mathematical modeling process.

Based upon the student level, interests, and local resources and professional availability, the system will also suggest opportunities for students and professionals to work together electronically and collaboratively to confront modeling challenges as a “affinity learning group”. Similar to electronic special interest groups or listservs, but more focused on a particular task or set of activities, these affinity learning groups will move forward together to share ideas and activities occurring on the system. In this way, students can tap the thinking of other students, as well as designated professionals.

Enhancing the ongoing dialogue between students and their teacher, students and other students, and students and other professionals, within such affinity learning groups is an important component targeted in the project design. Cognitive coaching and affinity learning as proposed in this project, have already been used successfully online in several classroom focused projects. For example, the Electronic Emissary Project, from the University of Texas at Austin, was able to match a wide range of students with scientists and other professionals to assist in answering questions on-line and for engagement in focused problem solving (Harris, 1996). This project would expand and add to such efforts by having the technology within the project assume a greater role in the facilitation of the structured dialogue process.

For extending student access to content expertise within the mathematical modeling process, the design team is targeting the development of something called an “Inquiry Garden” or “Knowledge Garden”, where questions can first be asked by a student to an interactive system. If the system can understand and respond to the student question, it does so based upon a stored database of knowledge related to the question, providing immediate feedback. If the student wants additional information, or doesn’t feel that the information is helpful, then the system can forward the student’s inquiry to a peer group of learners, or to the teacher, or to an outside expert or professional. As these individuals contribute new content based answers to the student, the system might store this new response, and index it as part the existing knowledge stored within the system related to that type of question. Such an extended and systematic inquiry process should help the student organize their thinking a bit, and efficiently tap the important resources represented by the thinking and feedback of others, as they work through a challenging mathematical modeling activity. For this important design feature of the project, the project is building upon the previous work of Henninger (1997, 1996), one of the PI’s of the project.

Design Principle 5) The adaptive learning system will help with the ongoing assessment of student understanding, through a systematic use of embedded assessments, as well as student self-assessment.

The systematic assessment of student understanding is a very important piece of the interactive technology being planned within the project. As educational technology continues to rapidly advance, new assessment opportunities and techniques are surfacing based upon these new technologies (Baker & O’Neil, 1995). Within this project, the use of assessments which are carefully integrated into the instructional environment, or “embedded” are targeted within the project design. For example, in an instructional activity where a student uses a spreadsheet to examine patterns of data within the mathematical modeling process, the system might record what variables the student is using within the spreadsheet, and perhaps the formulaic relationships between the variables. If key variables, or relationships have not been identified by the student, then additional instruction may be needed.

Drawing upon the potential of new technologies to store and index student information, an on-line student portfolio, or related student assessment profile, is targeted as a design feature. There is indeed a rich context of potential assessment information which often exists within an on-line or technology based learning environment, and the technology itself can indeed be a very useful tool in the organization of such information (Mathies, 1995). In fact, assessment variables which might be stored within the context of such a student profile are quite numerous. These variables can include a wide variety of student performance information. Some examples include the quality of the questions asked within electronic dialogue with the teacher or peers, the speed of response within a simulation environment, the approach used to set up on-line experiments to test data, the content information self-selected by the student to review. Student self-assessment can also be very rich within this context, and students might reflect periodically upon their own levels of understanding.

It is recognized within the design process being undertaken in the project, that any assessment will of course have some inherent degree of error in examining or predicting student understanding. An important component of the current prototype design planning in the project then is to try to identify potential “nodes” of student understanding, or related “nodes” of student misconception, within the content area of acceleration, based upon previous work of researchers in this topical area. We have defined a “node” to essentially be a key point within a student’s understanding related to a particular content area or process. For example, within the acceleration content area, such a node may be something as simple as recognizing that the student now understands that velocity

is changing over a period of time. It is recognized that some student presence at a particular node of understanding may initially be less defined or “fuzzy”, until the system has adequate data from the student to determine whether a particular level of understanding or “node” has been truly achieved.

Design Principle 6) The adaptive instruction will assist students in a systematic learning process, by carefully targeting instruction based upon their current levels of understanding.

The design of the project seeks to ensure that students will work from current levels of understanding (or achieved nodes as mentioned earlier), and are able to access content information as they are ready for it. The ability to move easily and systematically through content is an important component of any successful online instructional endeavor, and particularly when faced with the mathematics discipline (Harvey and Charnitski, 1998). For this design component, the project is building upon previous successes and expertise already established within the CLASS project (Communications, Learning, and Assessment in a Student-centered System) underway at the University of Nebraska at Lincoln.

Design Principle 7) The adaptive learning system will strive to assist students in the learning process, by acting as a non-threatening coach or assistant, which patiently helps them clarify their thinking process, examine possible approaches to the problem, and test possible solutions.

Within the design philosophy and process being undertaken in the project, student control is perhaps the most important design feature being incorporated. The vision for the project is one in which the student helps initiate, monitor, and direct their own learning process. The independent nature of the mathematical modeling process makes this a key design feature needed for any system which strives to assist in the modeling process (Smith, 1997). In fact, as described by Smith, the first independent run through the modeling process is often the most difficult. The education design within this project is paying particularly attention to this typical difficulty. Thus, the project seeks to assist students in learning how to initiate a mathematical modeling endeavor by helping assist their choice of modeling subtask, presenting the subtask appropriately, and delivering their the appropriate tutoring, and relevant instructional intervention, as needed. Facilitating such a systematic control by the student is also one the most difficult design challenges that we are facing in the project.

The Challenges of Building a Prototype System

It is indeed a daunting task to build an interactive and technology-based instructional system that truly follows the seven educational design principals set out by the team, and described in this paper. Each principal is itself an individual design challenge, and inherent with its own set of individual challenges when trying to be operationalized within the context of one or more components of a working system. However, the design team is building upon a solid foundation of earlier work, a commitment to innovative instruction and learning, and an ongoing dialogue with numerous colleagues. We are in essence seeking to conceptualize and examine how such system might contribute to all areas of the achievement cycle in mathematics education: curriculum, assessment, instruction, and learning (Glatthorn, Bragaw, Dawkins, & Parker, 1998); a cycle which is becoming all the more important with the growing commitment to standards based curriculum and related student achievement within our country.

A Final Thought: The Importance of Work in This Area

As an instructional team, we have found that mathematical modeling is indeed an appropriate topic for the content and focus for the design of a new adaptive instructional system. It is difficult to teach mathematical modeling in traditional ways, and the importance of mathematical modeling within the mathematics curriculum is well recognized, and seemingly growing. In addition, mathematical modeling is a natural outgrowth of the reform efforts of many committed organizations, such as the National Council of Teachers of Mathematics, and the

Mathematical Association of America. Although daunting, the design task we are addressing is also a very engaging and interesting one, and we are learning something almost every day about how students learn, and how we might better support them in that learning process. There is little doubt that good mathematical modeling instruction takes considerable effort by both the teacher and student; and that correspondingly, the development of instructional systems to support such a complex process will also take considerable design effort before a workable prototype or clear components to that prototype can emerge. However, it is also clear that in this situation, the design effort and energy to be expended is well worth it, as we seek to understand and help contribute to the exciting learning opportunities represented by new technologies, and new approaches to the learning process.

References

- Baker, E., O'Neil, H.F. (1995). Computer technology futures for the improvement of assessment. *Journal of Science Education and Technology*, 4(1), pp. 37-45.
- Corbat, M. (1985). The impact of computing technology on school mathematics: Report of an NCTM conference. *Mathematics Teacher*, 78 (April, 1985), pp. 243-250.
- Glatthorn, A.A., Bragaw, D., Dawkins, K., and Parker, J. (1998). Performance assessment and standards-based curricula: The achievement cycle. *Eye on Education*: Larchmont, NY. ISBN 1-883001-48-X.
- Harris, J. B. (1996). The electronic emissary: stories of expert-student mentoring in cyberspace. *The Proceedings of the National Educational Computing Conference 1996*, Minneapolis, Minnesota, pgs. 168-169.
- Harvey, F.A. and Charnitski, C.C. (1998). Improving mathematics instruction using technology. *Proceedings of selected research and development presentations at the National Convention of the Association for Educational Communications and Technology*, St. Louis, Missouri. ED 423837.
- Henniger, S., Lu, C., Faith, C. (1997). Using organizational learning techniques to develop context-specific usability guidelines. *Proceedings of the symposium on designing interactive systems (DIS 1997)*, Amsterdam, pp. 129-136.
- Henniger, S. (1996). Building organization specific infrastructure to support CASE tools. *Journal of automated software engineering*, 3(3/4), pp. 239-259.
- Kozman, R. and Grant W. (1995). Year 2005: using technology to build communities of understanding. In *Future Visions*, a report of the Office of Technology Assessment of the U.S. Congress, OTA-BP-EHR-169, pp. 121-145.
- Mathies, B.K. (1995). Technology and portfolios in the future. Chapter 5 in a guidebook entitled: *Portfolios in the Curriculum and Beyond* (Lead Author: D. Cole). ED 389759.
- Ostler, E. and Grandgenett, N. (1999). *Mathematical Modeling Activities: A Primer for Grades 5-12*. Midwest Educational Technology Services: Omaha, NE. ISBN 0-9667419-0-0.
- Richards, Barker, Meng Tan, Hudson, and Beachman. (1997). Knowledge sharing through electronic course delivery. *Innovations and Training International*, (34), 1, pgs. 3-9.
- Smith, D.N. (1997). Independent mathematical modeling. *Teaching Mathematics and its Applications*, 16(3), pp. 101-106.
- Swetz, F. and Hartzler, J.S. (1991). *Mathematical modeling in the secondary school curriculum*. The National Council of Teachers of Mathematics: Reston, Virginia. ISBN 0-87353-306-2.